**HOMEWORK 1- SOLUTION**

**Ans 1. DONE**

**Ans 2. FALSE**

Taking an example.

M prefers W to W’

M’ prefers W’ to W

W prefers M’ to M

W’ prefers M to M’

This is also a stable matching, but no such pair exists in which m has w,on his first preference, and w has m on her first preference simultaneously.

So the statement mentioned that in ever stable matching problem there is a stable matching containing a pair (m,w) with each being on each other’s preference list , being FALSE.

**Ans 3. TRUE**

Suppose we take a problem of two men and two women. Let a free man say M, goes and proposes W (who is highest on his list) and she accepts and the couple gets engaged. Now another free man, say M’ goes and proposes to the person who is on top of his list, who turns out to be W as well. He goes and proposes her, and she accepts this person as it turns out he is on top of her list as well. Man M turns single and proposes to the girl next on his list.

So the matching (m’,w) is done, who have each other as the top priority.

**Ans 4. TRUE**

In a consensus optimal stable matching , we compare the system of proposals by 2 versions. 1 by man proposing and the other by woman proposing. And if they are same, then the system shows consensus optimal stable matching which is unique in nature.

Eg: M: W, W’

M: W’,W

W: M, M’

W’:M’,M

The matching done will end up in result: (M, W) and (M’, W’) [ as they are first preferences of each other and if there is any abruptness , it will end up in instability and the system will not be unique.

**Ans 5. Not Always.**

Let’s Suppose the following case:

A: c,b,d

B: c,a,d

C: a,b,d

D: b,a,c

In this case, all three a, b , and c have placed d as their last preference. But one has to stay with d as the size of the room have maximum of 2 occupancy. So one of the three will end up with d thus , proving the statement wrong, as a,b,c will prefer each other to d and would be wanting to change the room eventually.

**Ans 6.** Eg of TV shows:

Let the television networks have 2 channels each and we compare the ratings of them

Network A: {a, a’} with rating being : { 50,100} resp

Network B: {b,b’} with ratings being: { 30, 150}resp

Now if we consider the set( a,b) and compare the ratings : { 50,30}

So network B would like to switch the channel to b’ to get more slots.

So now the set becomes: (a,b’) with the ratings being:{ 50,150}

Now as the channel a rating is < channel b rating, so network a will like to switch its channel to a’

And the process goes on, and thus leads to a unstable network system with unstable pair of schedules in the network systems.

**Ans 7.**

Algorithm: S students, H hospitals

Initially all s belong to S and all h belongs to H

While a hospital proposes an offer of joining to student s,

Let s being the highest preference of the hospital list

If s is free

Student s accepts the offer and joins hospital s

If s is not free, but prefers h’ to h

Student s rejects the offers

But if s prefers h to h’,

Student s accepts the offer and hospital h’ has a vacant spot

End if

End if

Endwhile

**First instability:**

We assume two students s, s’ and a hospital h. It is said that h prefers ( s’, s) which means it would have offered the job to s’ before it offered it to s and as s’ is free , will accept the job offer leading to the fact that s’ is not free anymore. Thus contradicting the statement.

**Second instability:**

Two student s and s’ and two hospitals h and h’.

H prefers s’>s , thus would have offered the job to s’ earlier as compared to s.

And as s’ also prefers h>h’ , will accept the job irrespective of the current status of being free or committed to another hospital.

Thus contradicting the fact that s is assigned to h.

**Ans 8.** Almazo Wilder – Nelly Oleson

M’ ----------- Laura Ingles

M-------------W

Initially AW- NO, and the system is stable. And then there is change in preferences and AW proposes to LI

If AW>M’ in Li list,

Then LI accepts

And M’ gets free.

Else LI rejects, LI ----- M’ and AW gets free and propose to next person in the list

Endif